Engineering Mechanics: Dynamics

Homework: Week 3 (11.07.2005–14.07.2005) | Due: 20.07.2005

1. Force-mass-acceleration method for a single particle: Please solve the following problems from Hibbeler.

(13-63) Think about this problem the next time you go over a hill in a car or roller coaster and get that queasy feeling in your stomach. Why might that feeling occur in those situations?

(13-67) This problem is relevant in light of all the roving amusement parts that travel around Pennsylvania in the summer.

(13-71) This sort of problem is pertinent to those who design package handling equipment for FedEx and UPS.

2. Cam dynamics: A cam is a simple device used to convert a rotary motion into a reciprocating motion; it consists of a projection from a rotating shaft that contacts a lever or rod. See, for example, `<http://www.bartleby.com/65/ca/cam.html>`. Cams are used in engines, old mechanical computers, and many other machines. Perhaps the simplest cam is made by rotating a disc about a point located a distance r from its center. The disc has a radius R. Determine the position $\mathbf{x}(t)$, velocity $\mathbf{v}(t)$, and acceleration $\mathbf{a}(t)$ of the point of contact between the cam and the follower. The follower always contacts the disc at the topmost point of the disc. (I'll show you a drawing of the cam and follower in class.) Suppose that the follower can be modeled as a particle of mass m located at the point of contact between the rod and the cam. What force is necessary to keep the follower in contact with the cam?

3. Forces on piston: Take another look at the piston from the previous homework set. Treat the piston head as a particle of mass m.

(a) Draw a free body diagram (FBD) and a mass–acceleration diagram (MAD) for the piston head. Include any normal forces constraining the piston to the x axis, the force the fluid in the piston chamber exerts, and friction — characterized by a coefficient of friction μ — between the chamber walls and the piston. Assume that any difference between the static and kinetic coefficients of friction is negligible. The head of the piston has an area A, and the pressure of the fluid is P. Ignore gravitational forces.

(b) From your FBD, MAD, and equation for $\mathbf{a}_x(t)$, write down the equations of motion for the piston head.

(c) For constant pressure P, at what angle(s) will the forces in the rod be least/greatest? For constant pressure P, at what angle(s) will the normal forces on the piston head be least/greatest?

(d) Suppose that the mass of the piston head is $m = 2$ kg (assume for the moment that the masses of the rod and flywheel are negligible compared to the piston head). Is our assumption of ignoring the gravitational forces a good one?

4. Flyball governor: A flyball governor is a device that can be used, for example, to control the speed of a steam engine. For some background, read the introduction to the following paper I've placed on the course web site (you don't have to read the whole paper; it's not necessary and goes beyond the dynamics we'll cover in this course):

- M. Denny, Watt steam governor stability, Eur. J. Phys 23 (2002) 339–351.

I've adapted Figure 2 from that paper. The device consists of two balls of mass m that are connected by rods CB, CB', MN, and MN' to two sleaves CC' and NN'. The sleave CC' is fixed to the shaft AA' at a height h, but the sleave NN' is free to move up and down. Shaft AA', and consequently the two balls, rotates with some — not necessarily constant — angular speed ω (a gearbox is used to make ω proportional to the speed of the engine). There is a spring (not shown) of spring constant k and unstretched length z_0 that connects the support at A' to the sleave NN'. If ω increases, NN' rises, and the linkages V and W cause a throttle to slow the speed of engine, and vice versa if ω decreases. We won't worry about friction, air resistance, the linkages V and W, or the mechanics of the throttle for this problem. We'll also neglect the width of the sleaves, and the masses of the rods and sleaves.

(a) Set up a cylindrical coordinate system, and derive expressions for the position $\mathbf{r}(t)$, velocity $\mathbf{v}(t)$, and acceleration $\mathbf{a}(t)$ of one of the balls. Since the device is symmetric, once you've found the dynamics for one mass, the dynamics of the other is easy to figure out.

(b) With the help of an appropriate FBD and MAD, derive the equations of motion for the ball.

(c) Find the angular velocity ω at which sleave NN' maintains a constant height. Set $b = d$ at this point to simplify the math. Express your answer as a function of b, c, d, k, z_0, and the other parameters in the problem. If $m = 1$ kg, $b = c = d = 10$ cm, $h = 20$ cm, and $z_0 = 5$ cm, what spring constant k should you pick for the governor to rotate at a steady 200 rpm?

5. Numerical analysis of projectile motion: In class, we saw how to solve the differential equations you encounter in dynamics with numerical methods. Download the Mathematica notebook on ANGEL that deals with projectile motion I show you and answer the (not many) questions I have asked in it. Have fun and play around a bit with it. Check your answers to the projectile motion problems in the previous homework set, especially your intuitions about how air resistance affects the calculations. You will most likely find these numerical techniques very useful for your projects.

6. Dynamics applied to archaeology: Estimate the gravitational potential energy of the Great Pyramid at Giza[1], and the amount of work needed to construct the pyramid. Some contend that the pyramids are so large that the Egyptians could not have built them without using special techniques (e.g., large kites) or help (e.g., from extraterrestrials). Make reasonable assumptions about the manpower available to a pharoah and the amount of work required to build a pyramid, and from them compute an estimate of the time needed construction. Could the Egyptians have done it?

7. Swings: Solve problems 14-72 and 15-108 from Hibbeler. When you finish them, please explain in words how a child pumping her legs and alternately sitting upright and leaning back causes the swing to swing higher.

[1] See ⟨`http://www.si.edu/resource/faq/nmnh/pyramid.htm`⟩ for some background information.

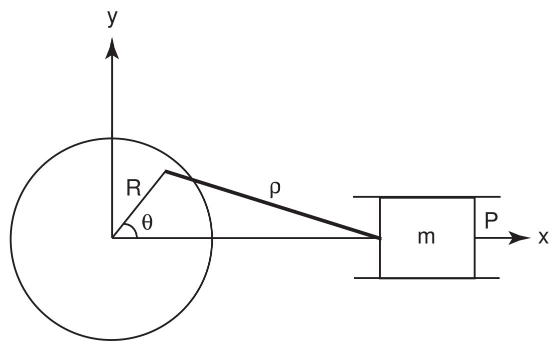

Figure 1: Schematic diagram of piston.

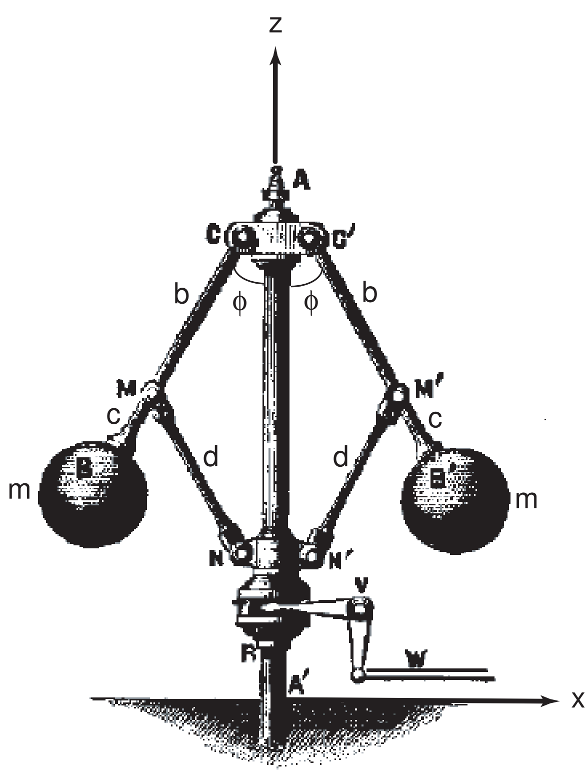

Figure 2: Diagram of a flyball governor.