Engineering Mechanics: Dynamics

Homework: Week 2 (05.07.2005–08.07.2005) | Due: 12.07.2005

1. Kinematics calisthenics: Please solve the following problems from Hibbeler.

(12-8) Assume a constant acceleration for the car (32.2 ft/s^2 toward the ground). What assumptions did you make in solving this problem?

(12-32) Comment on the applicability of this problem to the "four second rule" often taught in driver's education classes (i.e., when following a car on the highway, you should pass landmarks on the side of the road four seconds after the car in front of you). Is the rule a good one, and why or why not?

(12-36) Sketch $a(v)$ and $v(t)$.

(12-52) What assumptions did you make in solving this problem?

(12-136) Also, write down an expression for the jerk (hint: not your instructor's name) in rectangular coordinates too, i.e. in terms of x, y, and z and the Cartesian unit vectors. Note the difference in complexity in finding expressions for the acceleration and jerk in rectangular and cylindrical coordinates!

(12-153) Write expressions for the position, velocity, and acceleration of the car (as a function of time) in cylindrical coordinates, assuming that at $\theta(0) = 0$ and $z(0) = 36$ m.

Once you've finished these problems, you'll have a better understanding and intuitive feel for position, velocity, and acceleration in rectangular, polar, and cylindrical coordinates.

2. Cycloid kinematics: A disc of radius R_o rolls without slipping and with constant angular speed ω_o on a flat plane (see Figure 1).

(a) Derive equations for the coordinates $(x(t), y(t))$ of a point at a distance r from the center of the disc. At time $t = 0$ the point is at $x = 0$ and lies directly above the center of the disc. You can parameterize your answer in terms of an angle $\theta(t)$ measured from the y axis.

(b) From your answer to part (a), derive expressions for the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ of the particle.

(c) Plot the x and y components of the position, velocity, and acceleration of the point as a function of time using Mathematica or another computer program of your choice. For plotting purposes set $R_o = 1$ ft and $\omega_o = 1$ rps and compare the plots when $r = R_o/2$, $r = R_o$, and $r = 2R_o$. Before you do this, sketch out what you think the graphs will look like and then compare them with your computer plots.

Figure 1: Diagram of disc.

Note that when $r = R_o$, the path the particle takes is called a cycloid.

3. Running a yellow light: Jearl Walker, former author of the "Amateur Scientist" column for Scientific American magazine, poses the following problem[1].

[1] Jearl Walker, The Flying Circus of Physics with Answers, John Wiley & Sons: New York, NY, USA (1977).

Every driver will occasionally have to make a quick decision whether or not to stop at a yellow light. His intuition about this has been built up by many tests and some mistakes, but a calculation might reveal some situations where intuition will not help.

For some given light duration and intersection size, what combinations of initial speed and distance require you to stop (or run a red light)? What range of speed and distance allow you to make it through in time? Notice that for a certain range of parameters you can choose either to stop or not. But there is also a range in which you can do neither in time, in which case you may be in a lot of trouble.

Suppose that the yellow light is on for a time t_y, and that the car starts out at speed v_o a distance d from an intersection of width w. Assume that when the driver floors the gas pedal, the car can accelerate at a maximum acceleration a_a, and that the car can brake at a maximum acceleration a_b. Using these assumptions, answer Walker's questions. Draw a plot with d along one axis and v_o along the other, and mark off regions where the driver can accelerate safely through the intersection, brake before reaching it, or do neither. Once you've done this, plug in some reasonable values of the parameters for an intersection you are familiar with (e.g., the intersection of College and Allen streets in State College). Do your answers make sense? From this exercise, can you say anything about how speed limits and timing of yellow lights are decided? What considerations—both those arising from dynamics and not—are involved in such decisions?

4. Piston: Consider a piston that is constrained to move on the x axis, as shown in Figure 2. A rod of length ρ connects the piston head to a crankshaft at a point a distance R from the center of the crankshaft. The crankshaft rotates counterclockwise at a constant angular speed $\dot{\theta} = \omega$, and thus $\theta(t) = \omega t$.

(a) Find an expression for the position $x(t)$ of the piston head as a function of time t. Use the initial conditions $\theta(0) = 0$ and $\dot{\theta}(0) = \omega$. Check that your expression makes sense for the cases where $\theta = 0$ and $\theta = \pi$.

(b) Find the velocity $\mathbf{v}_x(t)$ and acceleration $\mathbf{a}_x(t)$ of the piston. Hint: you'll obtain the following expression for the acceleration:

$$\mathbf{a}_x(t) = a_x \mathbf{e}_x = -R\omega^2 \left(\cos\theta + \frac{\cos(2\theta)}{\left[\left(\frac{\rho}{R}\right)^2 - \sin^2\theta\right]^{\frac{1}{2}}} + \frac{\sin^2(2\theta)}{4\left[\left(\frac{\rho}{R}\right)^2 - \sin^2\theta\right]^{\frac{3}{2}}} \right) \mathbf{e}_x \,, \tag{1}$$

where $\theta(t) = \omega t$.

(c) To get an idea of the velocities and accelerations that the piston head can undergo (for example in a pump or internal combustion engine), set $\omega = 50\,\pi$ rad/s (i.e., 1500 rpm), $R = 5$ cm, and $\rho = 15$ cm. Plot x, v_x, and a_x for the piston head as a function of θ from 0 to 2π using Mathematica or another computer program of your choice. Express your answers for the acceleration both in units of m/s^2 and in g's (1 g $\approx$ 9.8 m/s^2).

When you finish this problem, I hope you'll have a better appreciation for how quickly parts move around inside an automobile engine.

5. Projectiles in a flat, airless world: A projectile — possibly a football, or an artillery shell — is launched from a height h_o with speed v_o at angle θ_o (measured from the horizontal). Neglect air resistance and assume that the earth is a flat, inertial reference frame. Treat the projectile as a particle.

(a) Find the range r of the projectile as a function of h_o, v_o, and θ_o. If $h_o = 0$, for what value of θ_o is the range maximized? If $h_o \neq 0$, is the range maximized by a higher or lower value of θ_o than when $h_o = 0$, and why?

(b) Suppose that, in your role as a quarterback or a gunner, you need to hit a target at a range r_o; the height h_o and initial speed v_o of the projectile are fixed. At what angle(s) should you launch the projectile to hit the target? What is the minimum value of v_o that is required to hit the target, assuming $h_o = 0$?

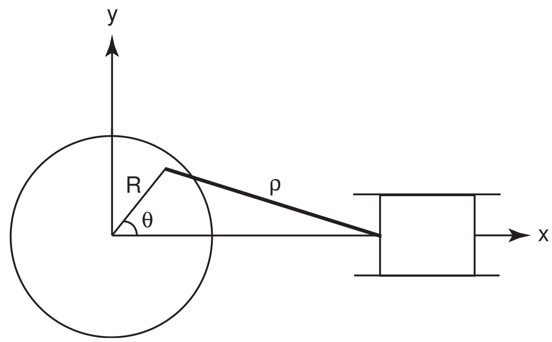

Figure 2: Schematic diagram of piston.

(c) Again assuming $h_o = 0$, what errors Δv_o and $\Delta\theta_o$ can be tolerated in v_o and θ_o, respectively, if the projectile is to hit the target at r_o—i.e. land somewhere between $r_o - \Delta r_o$ and $r_o + \Delta r_o$? Here Δr_o is the amount of error that can be tolerated—for a pass in football, this might be the length of the receiver's arms. Assuming reasonable values for v_o, θ_o, and Δr_o, make a numerical estimate of the errors Δv_o and $\Delta\theta_o$ that can be tolerated in order to complete a 50-yard pass down a football field.

(d) How would your answers to parts (a)–(c) change, if at all, if air resistance was included in your calculations (i.e., would they increase, decrease, or remain unchanged)? Why?

Keep this exercise in mind the next time you watch a Penn State game.